

# A $q$ -analogue of distance matrix of bi-block graphs

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## Abstract

A  $q$ -analogue of the distance matrix, referred to as the  $q$ -distance matrix, is obtained from the distance matrix by replacing each nonzero entry  $\alpha$  with the sum  $1 + q + \cdots + q^{\alpha-1}$ . This notion was introduced independently by Bapat, Lal, and Pati [1], and by Yan and Yeh [5]. A connected graph is called a *bi-block graph* if each of its blocks is a complete bipartite graph. In this paper, we derive explicit formulas for the determinant and the inverse of the  $q$ -distance matrix of bi-block graphs. These results both generalize the corresponding formulas for the distance matrix of bi-block graphs obtained in [3] and extend the results for block graphs in [4] to the class of bi-block graphs.

## References

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